Hydromagnetic dynamos in rotating spherical fluid shells in dependence on the Prandtl number and stratification

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Parameters

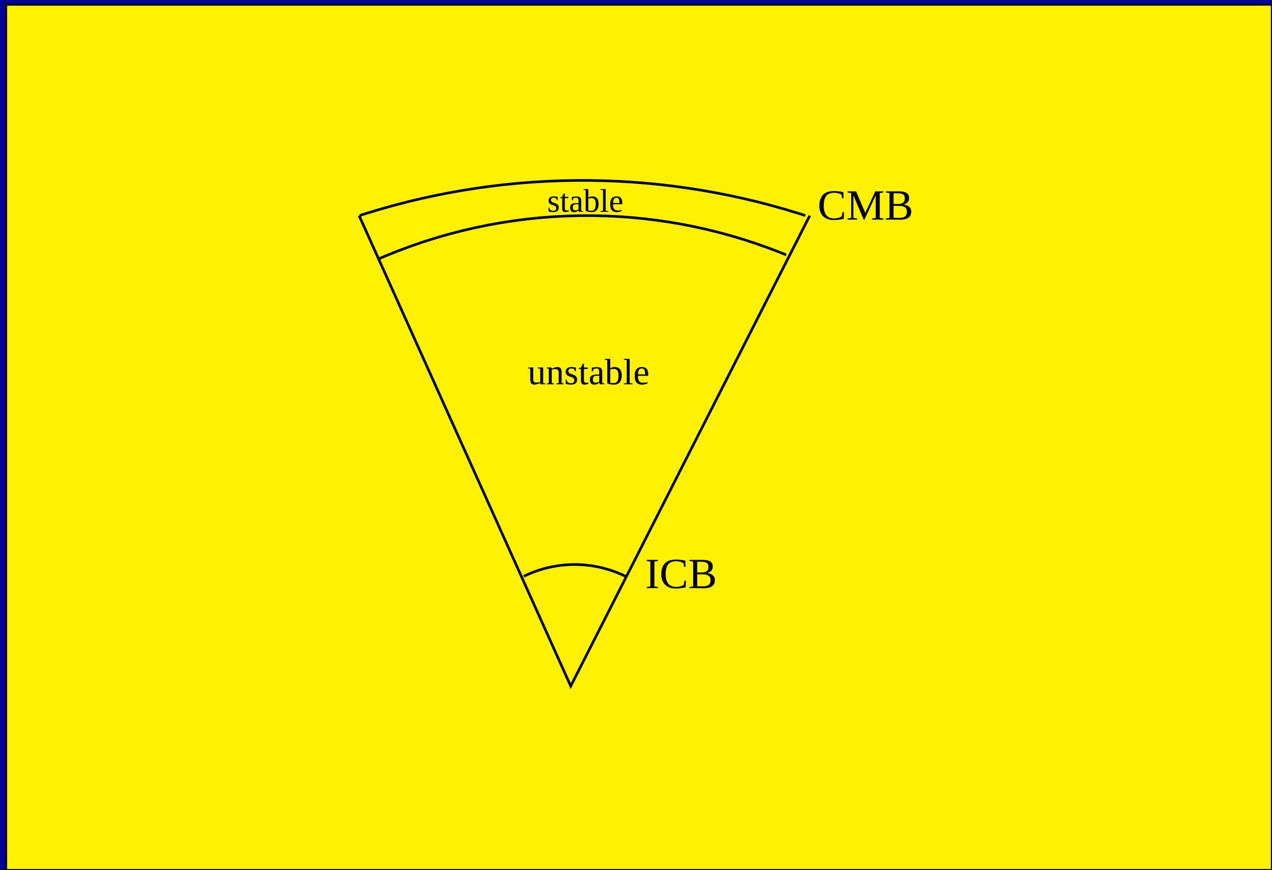
Parameter	Core	Dynamo models
R_a/R_{a_c}	$\gg 1 \ (R_a \sim 10^{30})$	$1-50$
E	$\mathcal{O}(10^{-15})$	$\mathcal{O}(10^{-7})$
P_m	$\mathcal{O}(10^{-7})$	$0.01-1$
P_r	0.2	$0.1-1$

$$P_{m_{min}} \simeq 450E^{3/4}$$

Christensen, Olson and Glatzmaier (1999)

Non-uniform stratification

Braginsky (1964)



10% of the shell is stably stratified and 90% unstably

Governing equations

$$E\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\,\mathbf{V} - \nabla^2\mathbf{V}\right) + 2\mathbf{1}_z \times \mathbf{V} + \nabla P =$$

$$= R_a \frac{\mathbf{r}}{r_o} T + \frac{1}{P_m}(\nabla \times \mathbf{B}) \times \mathbf{B},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{P_m}\nabla^2\mathbf{B},$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)\,T = \frac{1}{P_r}\nabla^2 T + H,$$

$$\nabla\cdot\mathbf{V} = 0, \quad \nabla\cdot\mathbf{B} = 0$$

Boundary conditions

- **the non-penetrating and no-slip boundary conditions for the velocity field at the rigid surfaces**

- **the constant temperature** $T_i = 1$ **and** $T_o = 0$ **at the inner (ICB) and outer (CMB) boundaries of the shell, respectively**

- **the CMB is electrically insulating, while the ICB is electrically conducting**

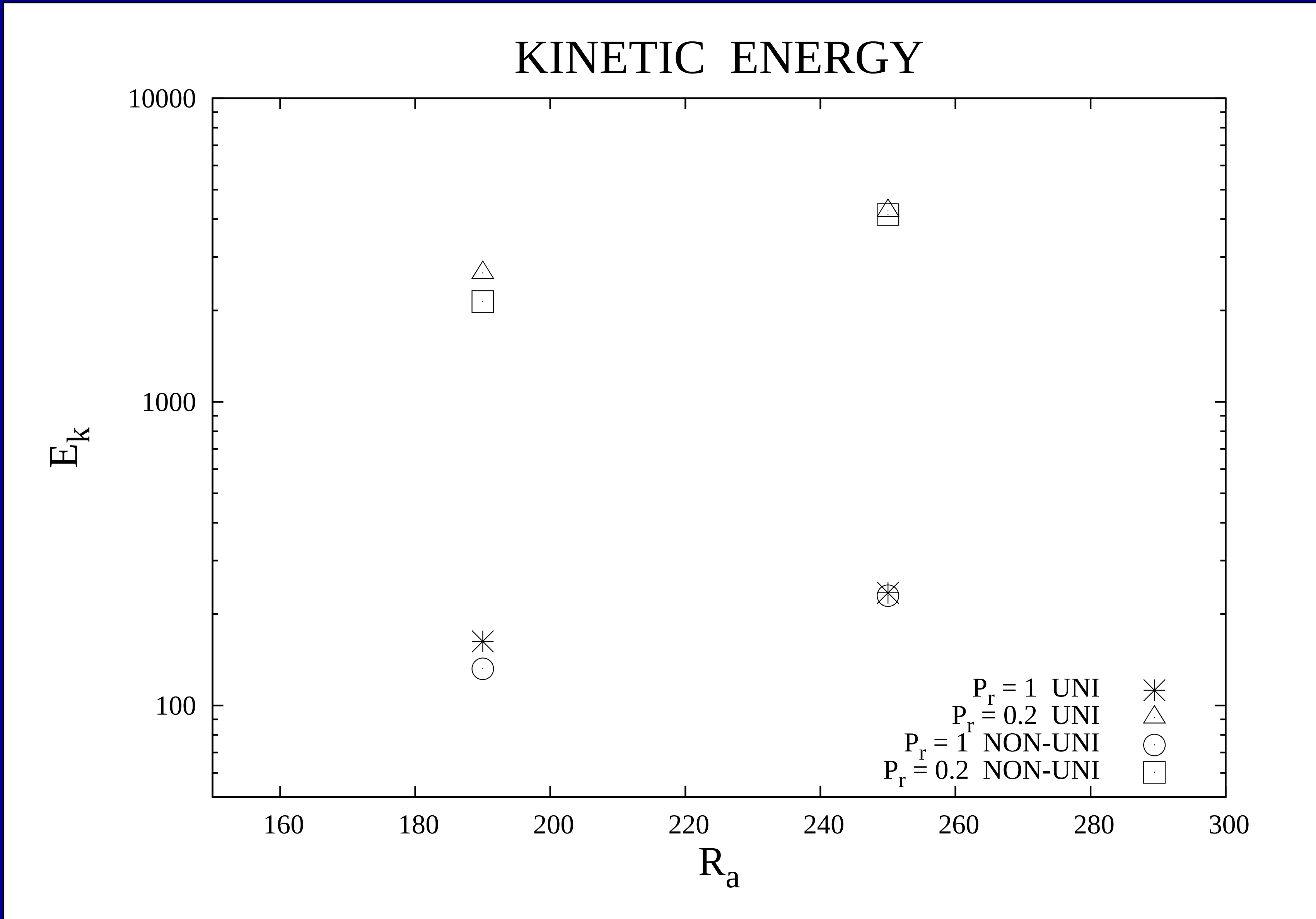
Case $E = 10^{-3}$

$E = 10^{-3}, \ \ P_m = 3 \ \ (P_{m_{min}} = 2.5), \ \ P_r = 0.2, \ 1,$

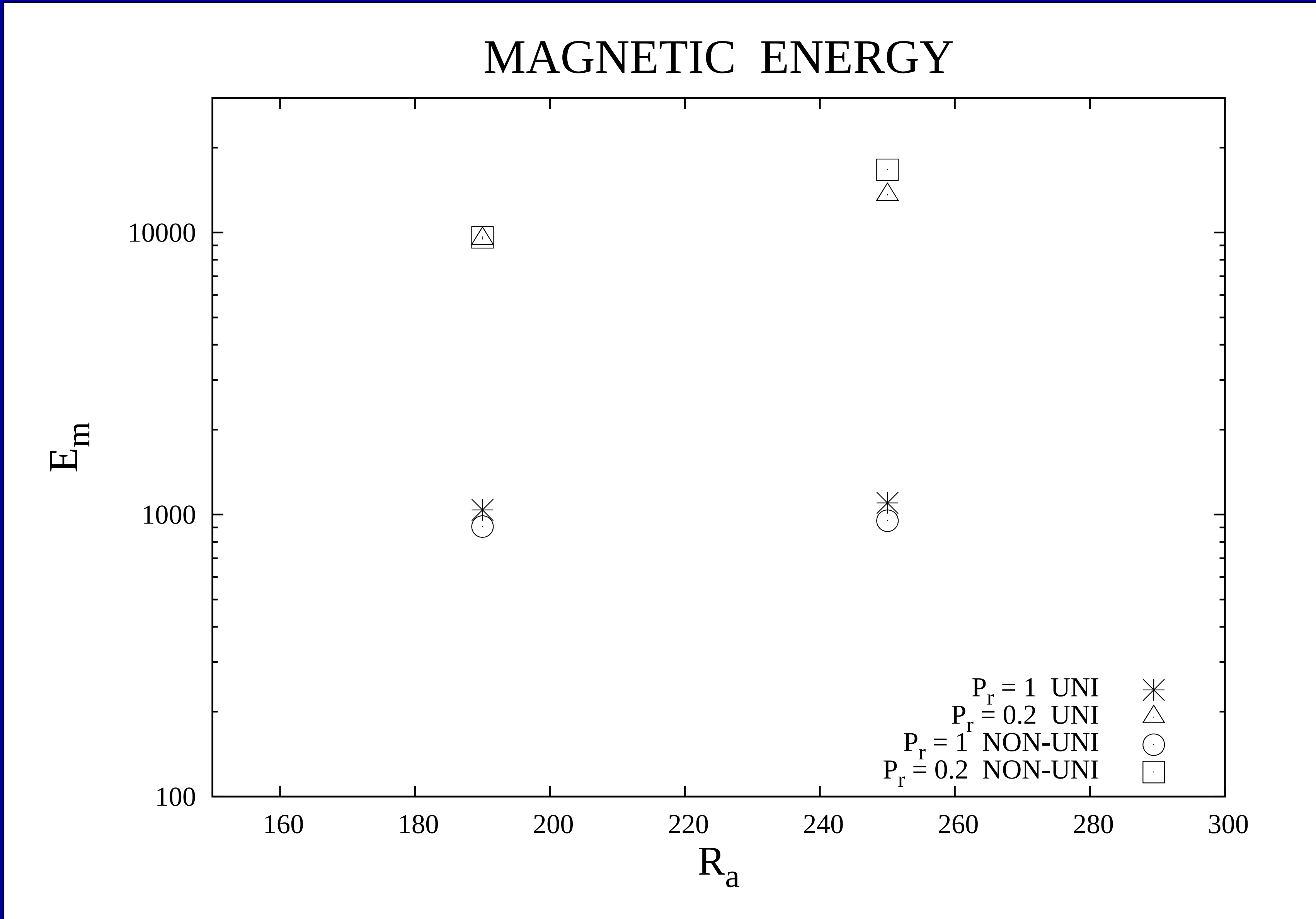
$R_a = 190, \ 250$

$$H = \begin{cases} -2.78 & \textbf{for} \quad P_r = 1 \\ -13.9 & \textbf{for} \quad P_r = 0.2 \end{cases}$$

MAG *http://www.geodynamics.org/cig/software/packages/geodyn/mag/*

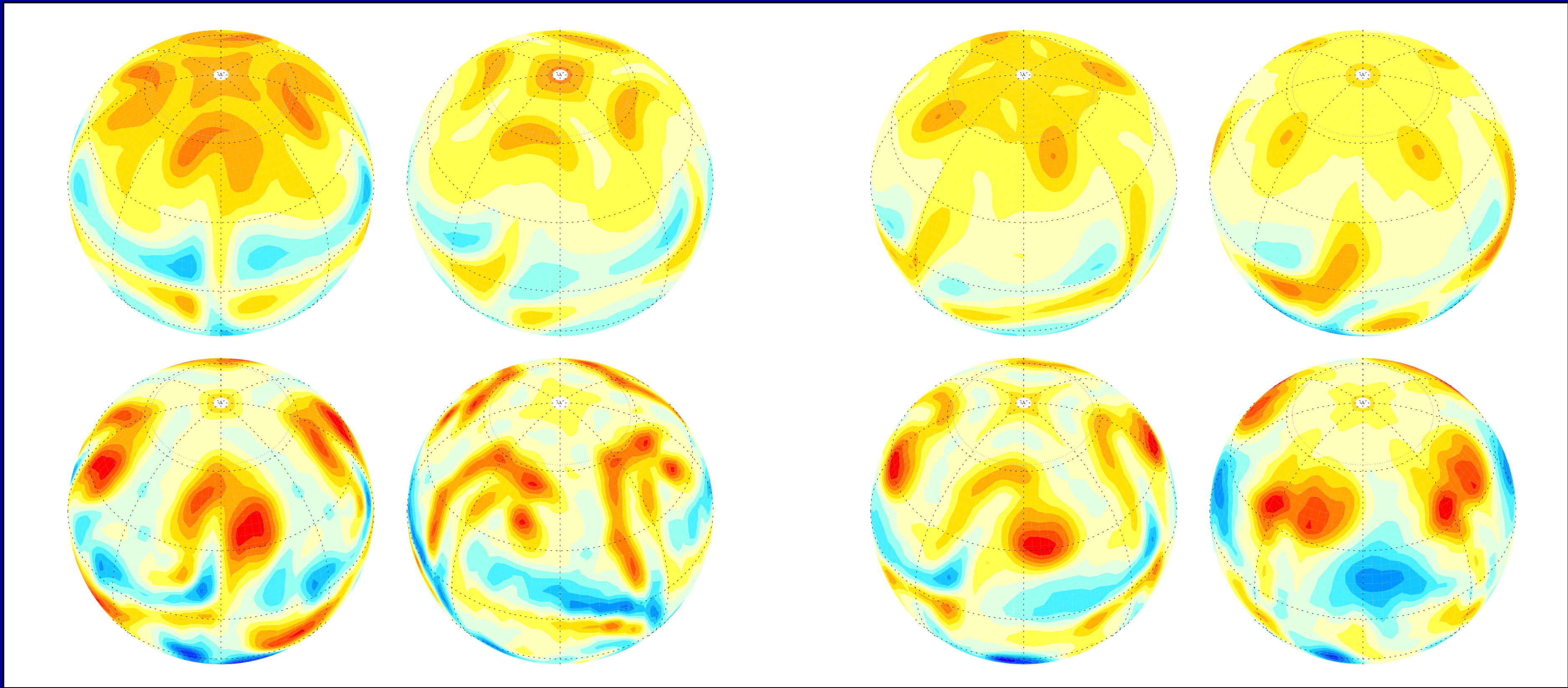

KINETIC ENERGY
10000
1000
100
E_k
160
180
200
220
240
260
280
300
R_a
P_r = 1 UNI
P_r = 0.2 UNI
P_r = 1 NON-UNI
P_r = 0.2 NON-UNI

$\tau = 10$


MAGNETIC ENERGY
E_m
10000
1000
100
160
180
200
220
240
260
280
300
R_a
P_r = 1 UNI
P_r = 0.2 UNI
P_r = 1 NON-UNI
P_r = 0.2 NON-UNI

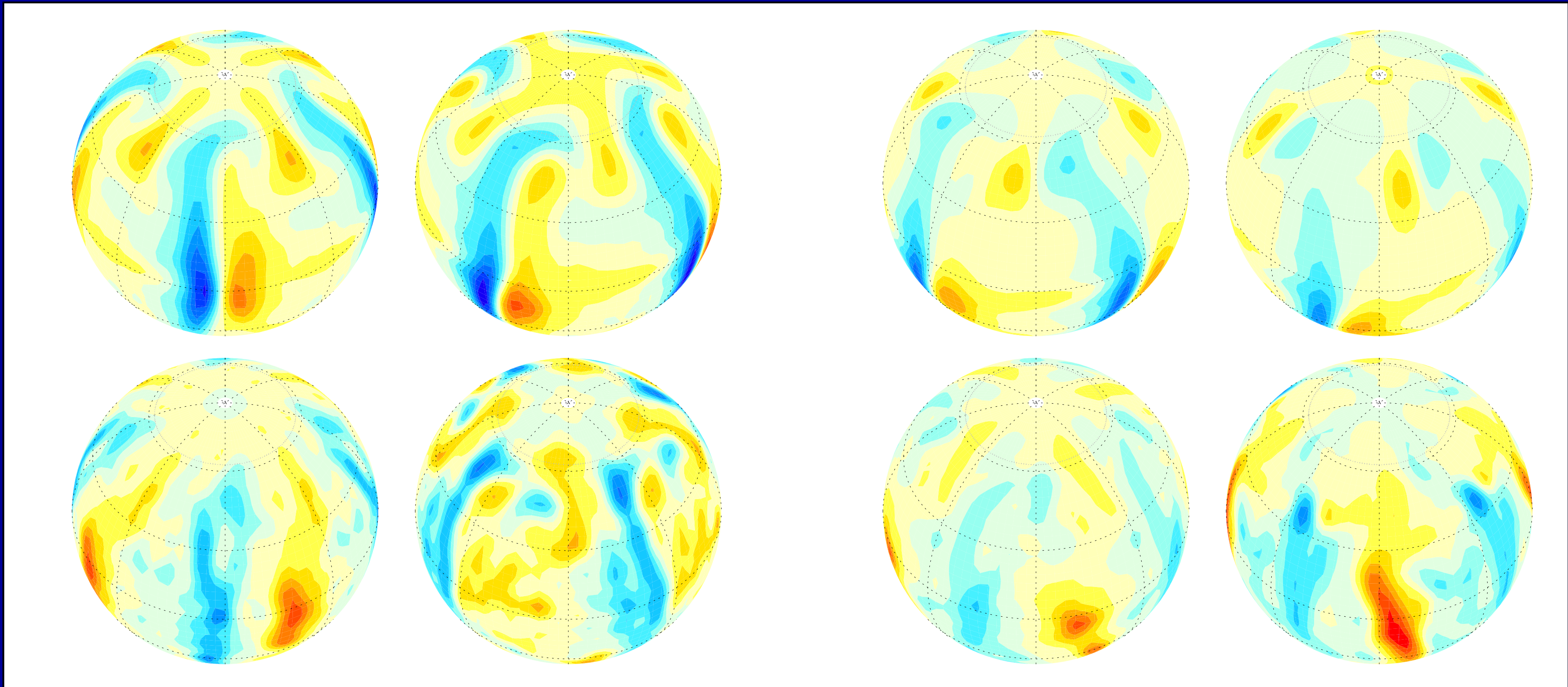
$\tau = 10$

B_r **at** $r = 0.85$



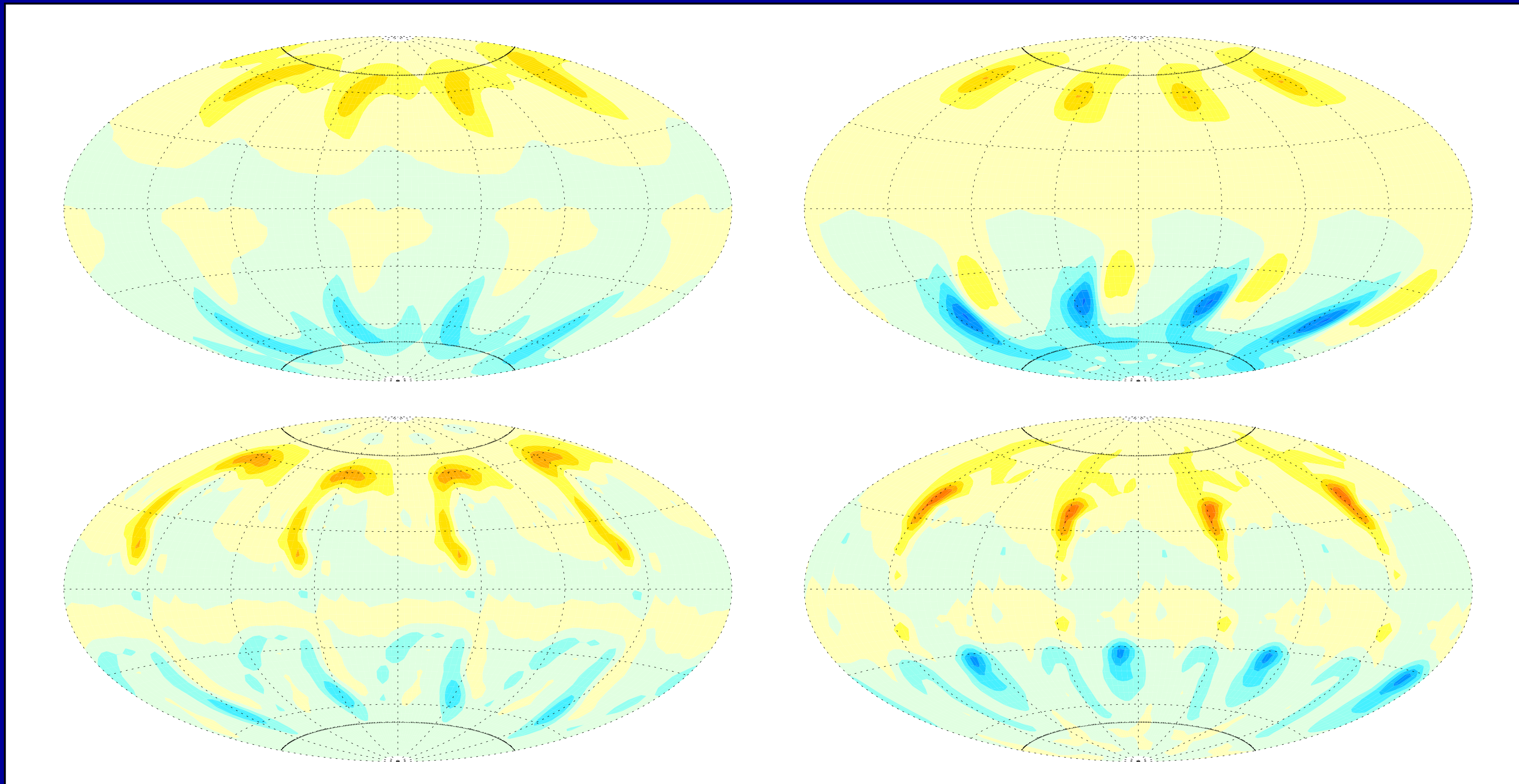
$$\begin{pmatrix} P_r = 1 & 190\,UNI & 250\,UNI & 190\,NON-UNI & 250\,NON-UNI \\ P_r = 0.2 & 190\,UNI & 250\,UNI & 190\,NON-UNI & 250\,NON-UNI \end{pmatrix}$$

V_r **at** $r = 0.85$



$$\begin{pmatrix} P_r = 1 & 190\,UNI & 250\,UNI & 190\,NON-UNI & 250\,NON-UNI \\ P_r = 0.2 & 190\,UNI & 250\,UNI & 190\,NON-UNI & 250\,NON-UNI \end{pmatrix}$$

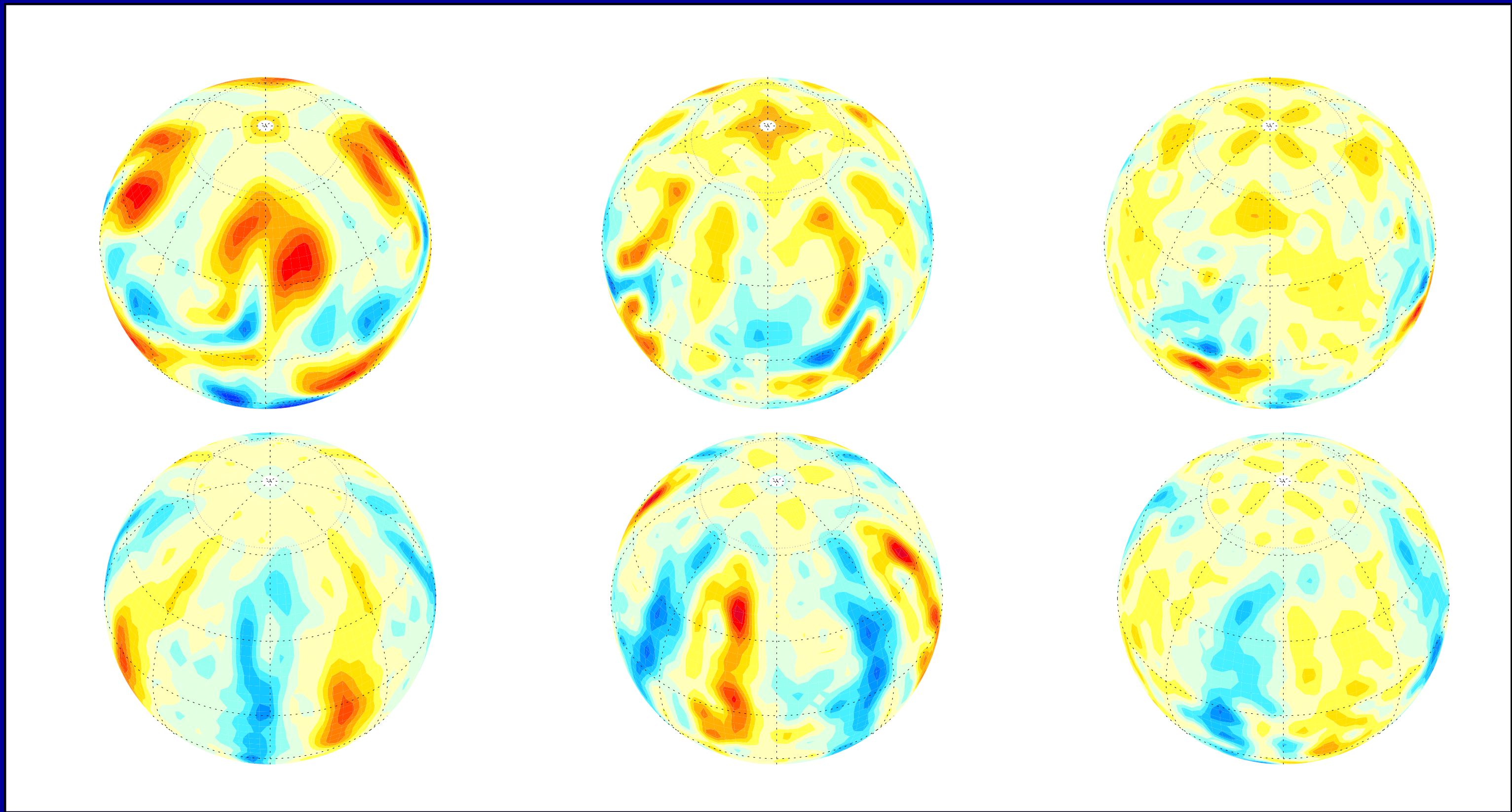
B **at** $r = r_o$



$$\left(\begin{array}{c|cc} P_r = 1 & 250\,UNI & 250\,NON-UNI \\ P_r = 0.2 & 250\,UNI & 250\,NON-UNI \end{array} \right)$$

B_r **and** V_r **at** $r = 0.85$

$P_m = 3,\ 15,\ 35 \quad P_r = 0.2$



$$\left(\begin{array}{c|ccc} B_r & 190\,UNI,\ P_m = 3 & 190\,UNI,\ P_m = 15 & 190\,UNI,\ P_m = 35 \\ V_r & 190\,UNI,\ P_m = 3 & 190\,UNI,\ P_m = 15 & 190\,UNI,\ P_m = 35 \end{array} \right)$$

Rossby number

	$R_o(P_r = 1)/R_o(P_r = 0.2)$
$R_a = 190$ **UNI**	**0.266**
$R_a = 190$ **NON-UNI**	**0.248**
$R_a = 250$ **UNI**	**0.235**
$R_a = 250$ **NON-UNI**	**0.236**

$$R_o = V/L\Omega = ER_mP_m^{-1}$$

Conclusions - stratification

- the influence of non-uniform stratification is for our parameters weak
- thin stably stratified region very slightly destabilizes the dynamo for $P_r = 1$, while for $P_r = 0.2$ either does not influence or very slightly stabilizes the dynamo
- we do not observe in our dynamos an eastward drift (*Stanley and Mohammadi (2008)*) of magnetic flux spots in the equatorial regions, it is always westward
- our stably stratified layer is not very strongly stratified, i.e. our model is perhaps characterized by the mild stable stratification

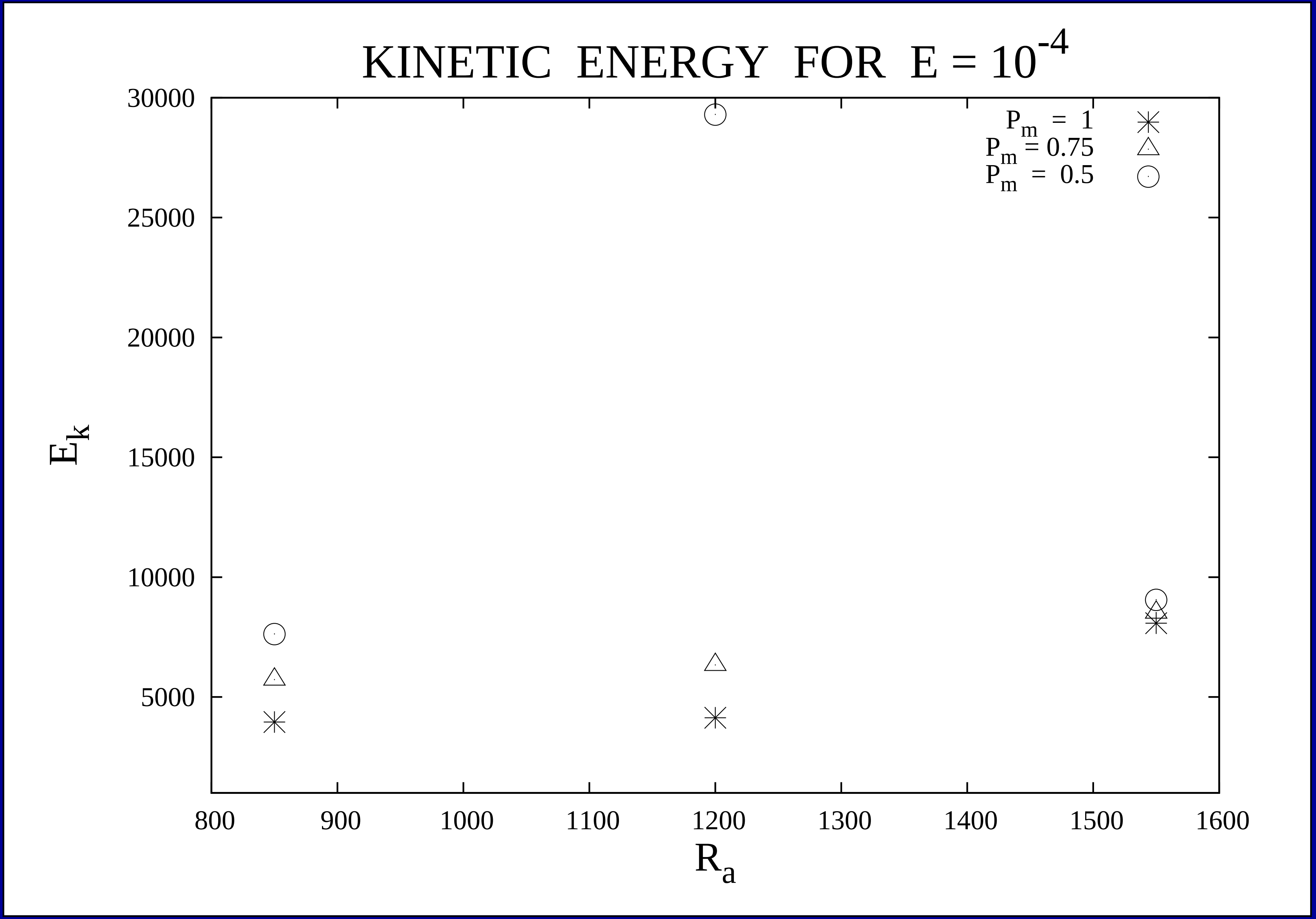
Case $E = 10^{-4}$

$E = 10^{-4}, \;\; P_m = 1,\, 0.75,\, 0.5 \;\; (P_{m_{min}} = 0.45),$

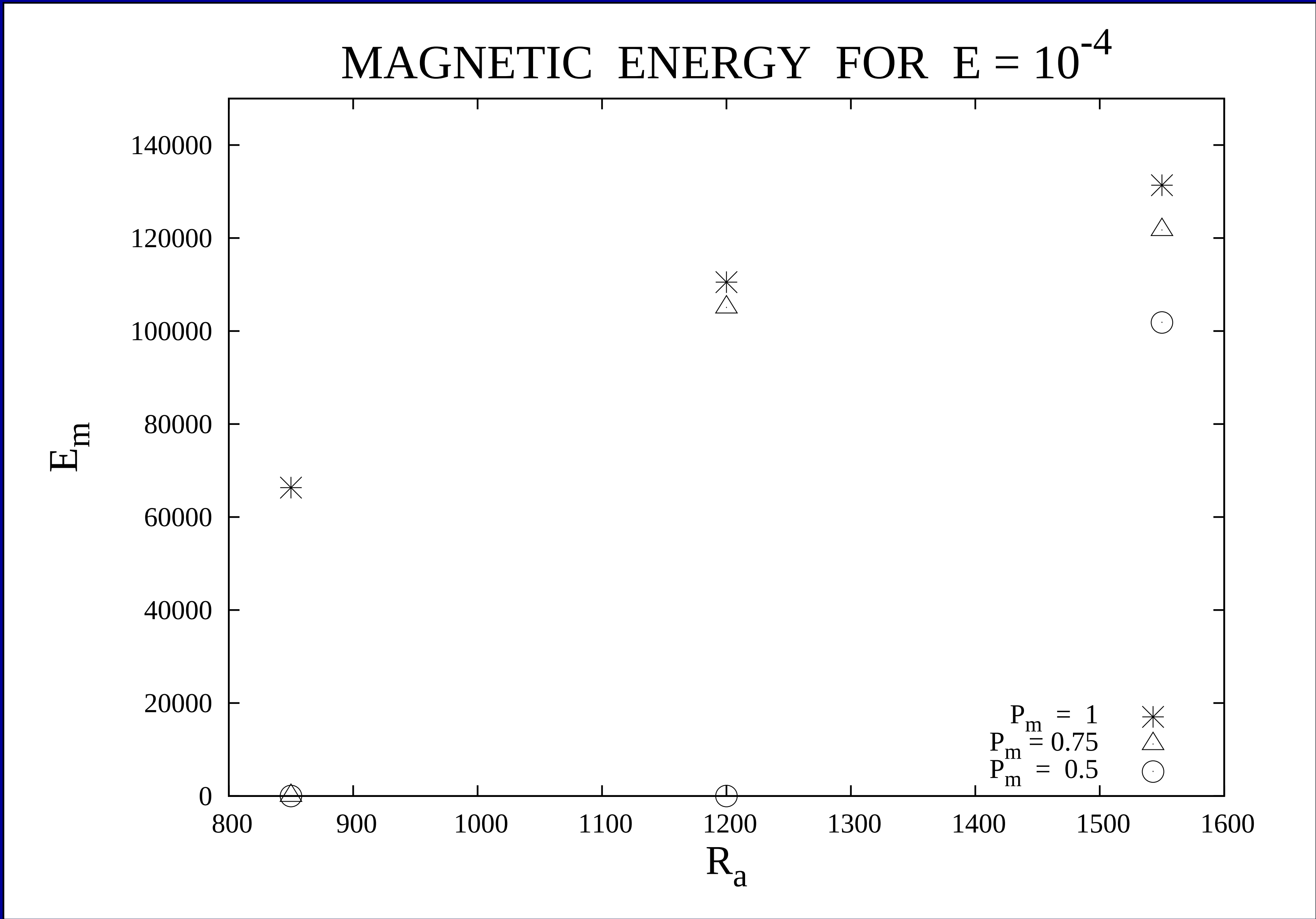
$P_r = 0.2, \;\; R_a = 850,\, 1200,\, 1550$

$H = 0$

PARODY and DMFI *Aubert, Aurnou and Wicht (2008)*


KINETIC ENERGY FOR E = 10^-4
30000
25000
20000
15000
10000
5000
E_k
800
900
1000
1100
1200
1300
1400
1500
1600
R_a
P_m = 1
P_m = 0.75
P_m = 0.5

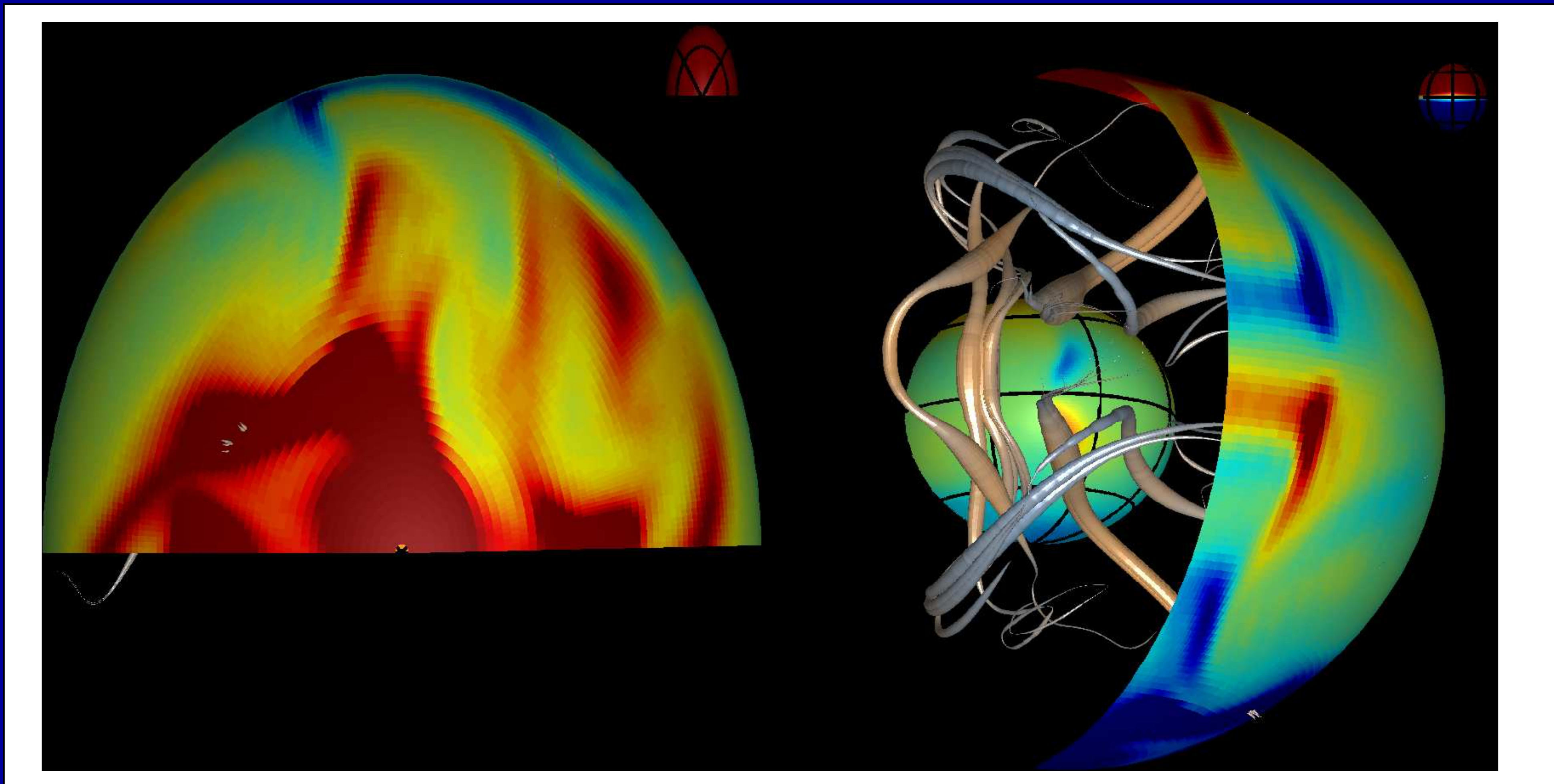
$\tau = 8$



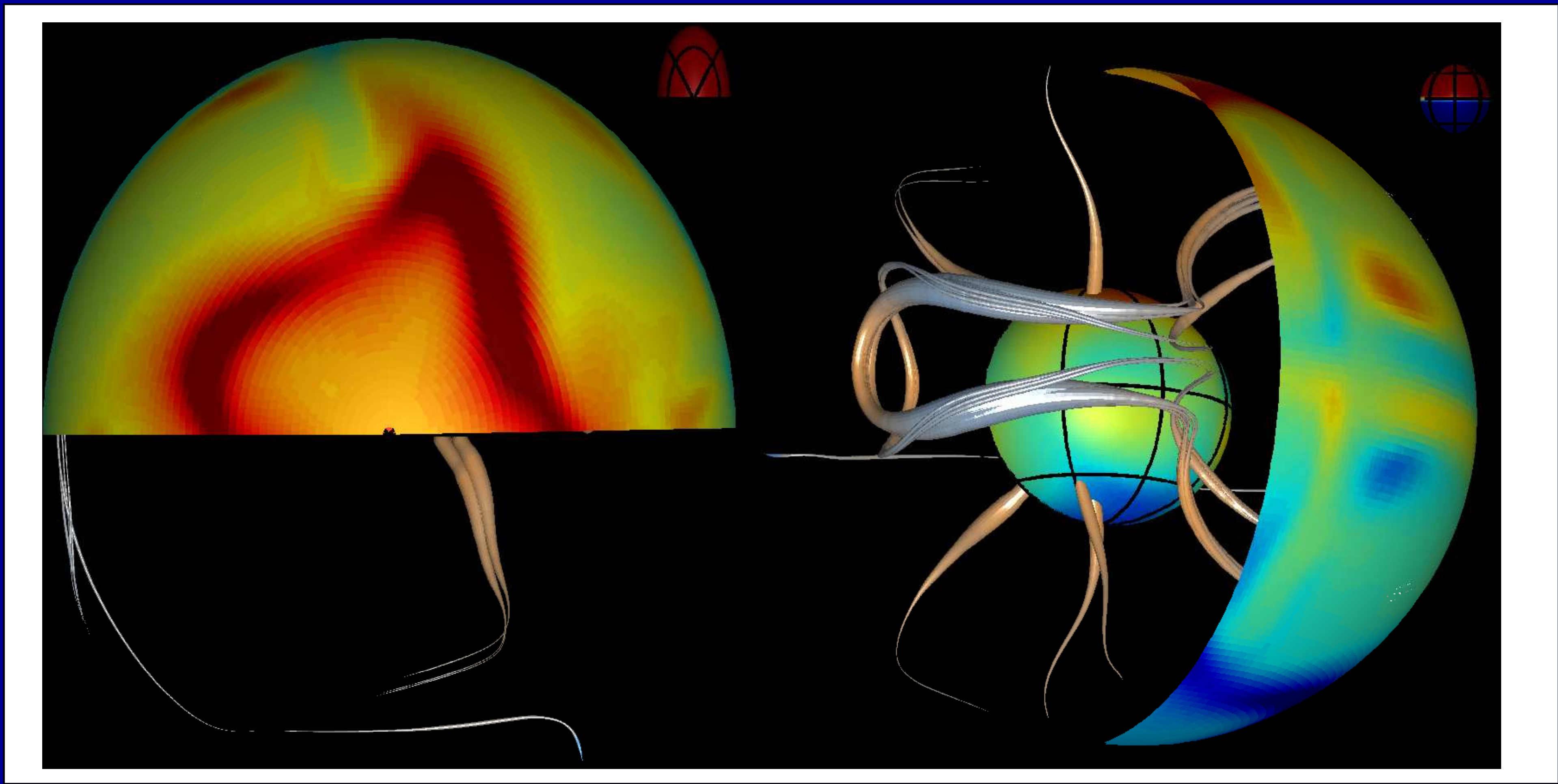
MAGNETIC ENERGY FOR E = 10^-4
E_m
R_a
0
20000
40000
60000
80000
100000
120000
140000
800
900
1000
1100
1200
1300
1400
1500
1600
P_m = 1
P_m = 0.75
P_m = 0.5

$\tau = 8$

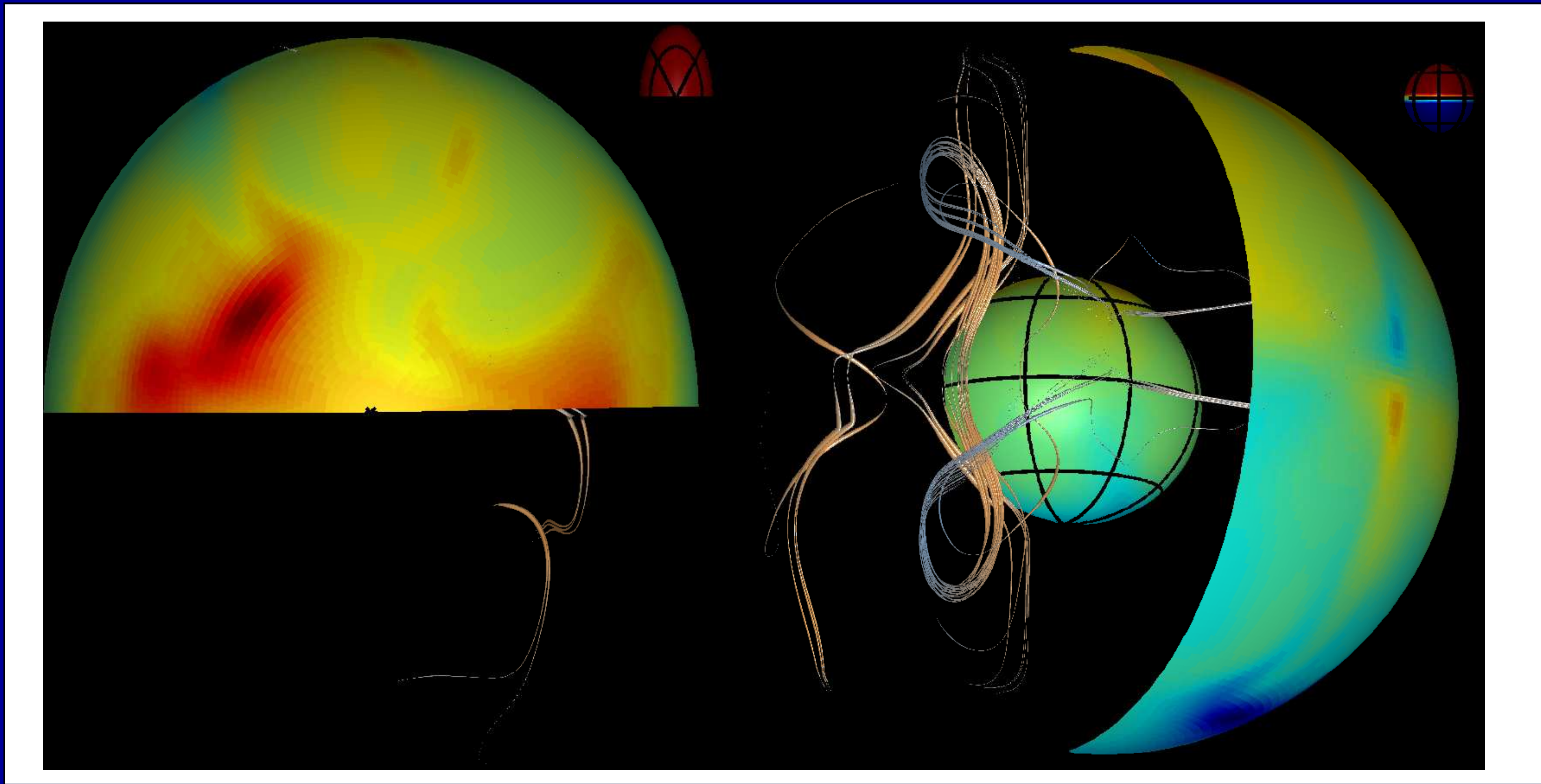
B_r **for** $R_a = 1550$ **and** $P_m = 1$



B_r **for** $R_a = 1550$ **and** $P_m = 0.75$



B_r **for** $R_a = 1550$ **and** $P_m = 0.5$



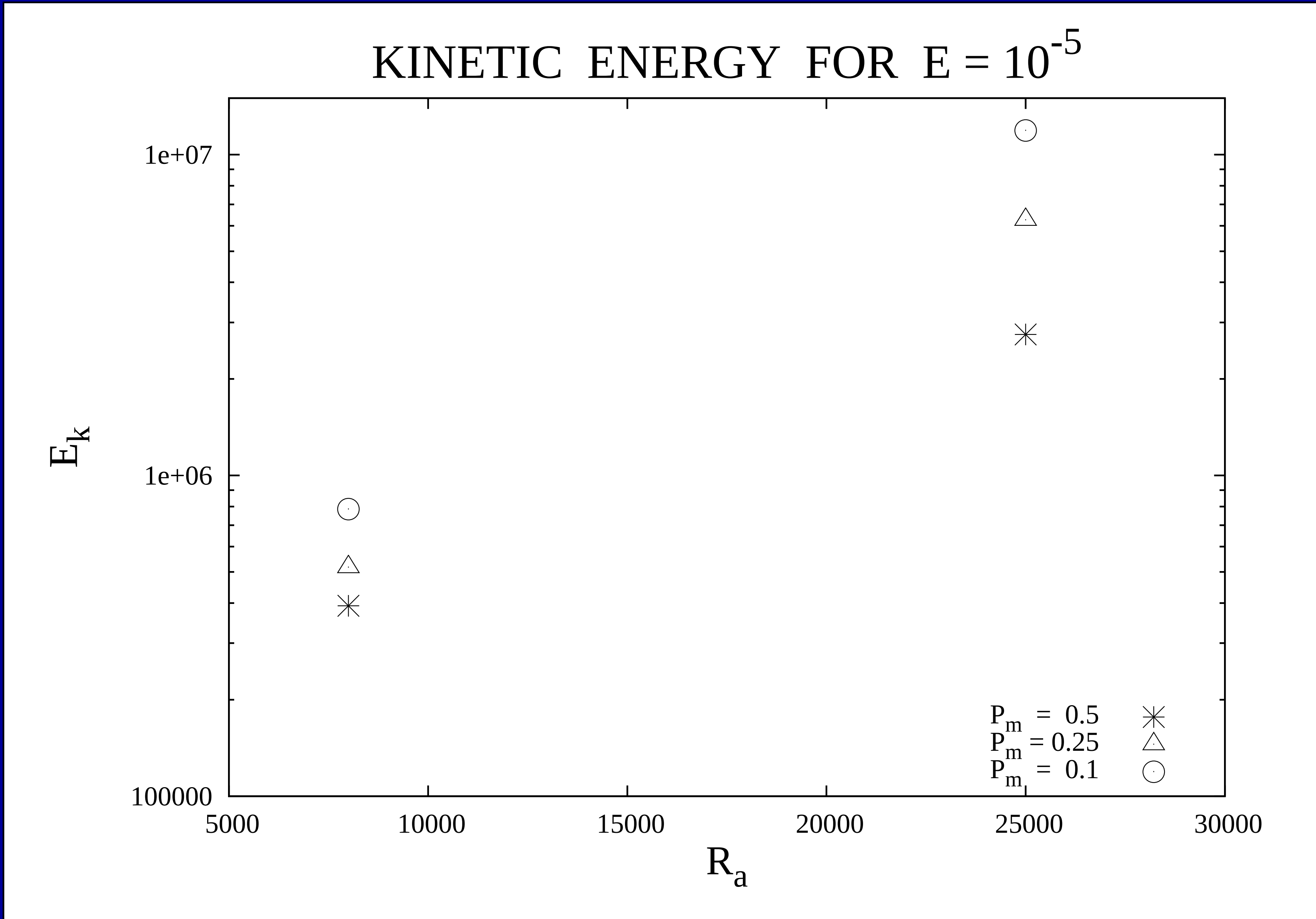
Case $E = 10^{-5}$

$E = 10^{-5}, \;\; P_m = 0.5,\, 0.25,\, 0.1 \;\; (P_{m_{min}} = 0.08),$

$P_r = 0.2, \;\; R_a = 8000,\, 25000$

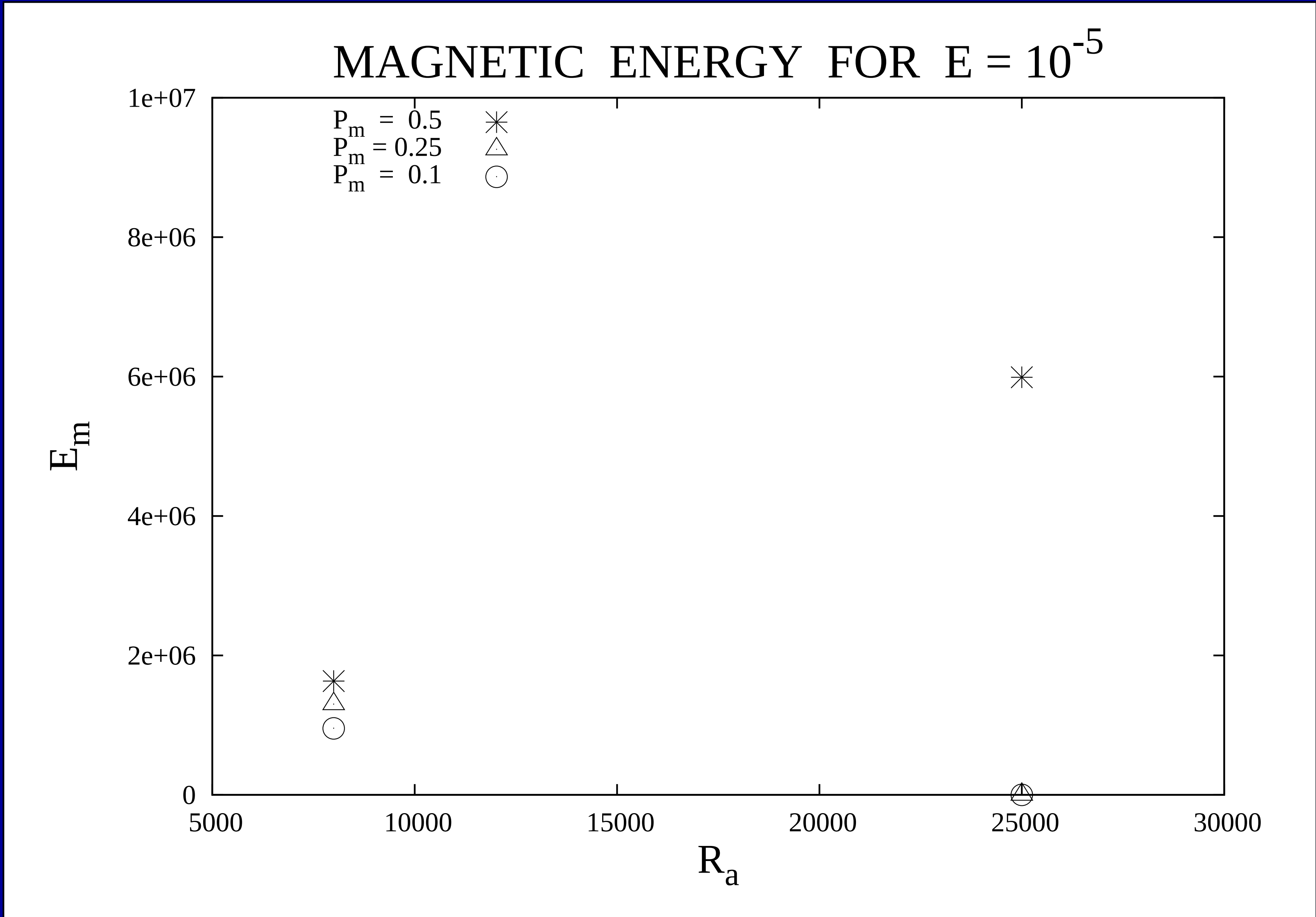
$H = 0$

PARODY and DMFI *Aubert, Aurnou and Wicht (2008)*



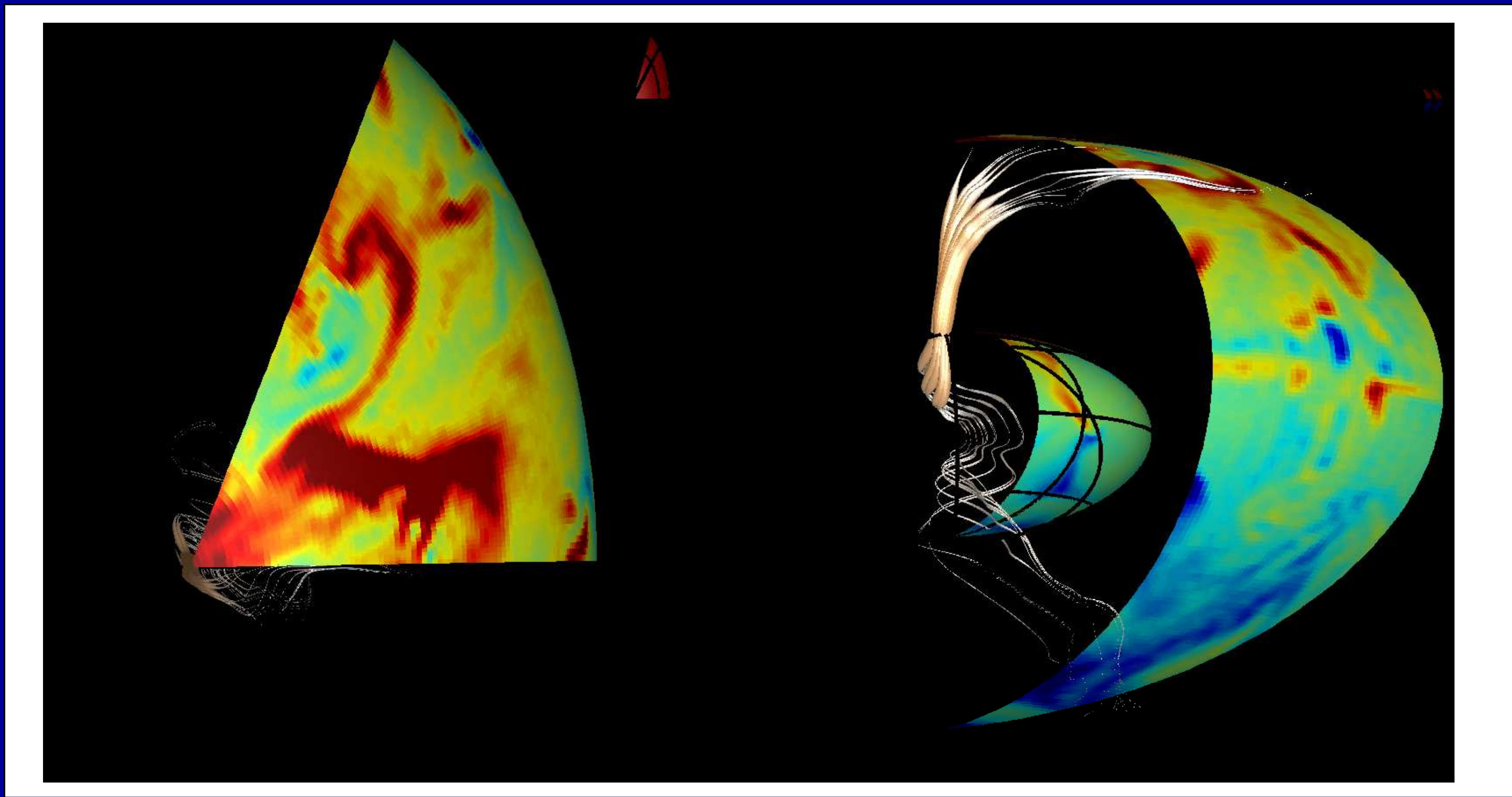
KINETIC ENERGY FOR E = 10^-5
1e+07
1e+06
100000
E_k
5000
10000
15000
20000
25000
30000
R_a
P_m = 0.5
P_m = 0.25
P_m = 0.1

$\tau = 5$

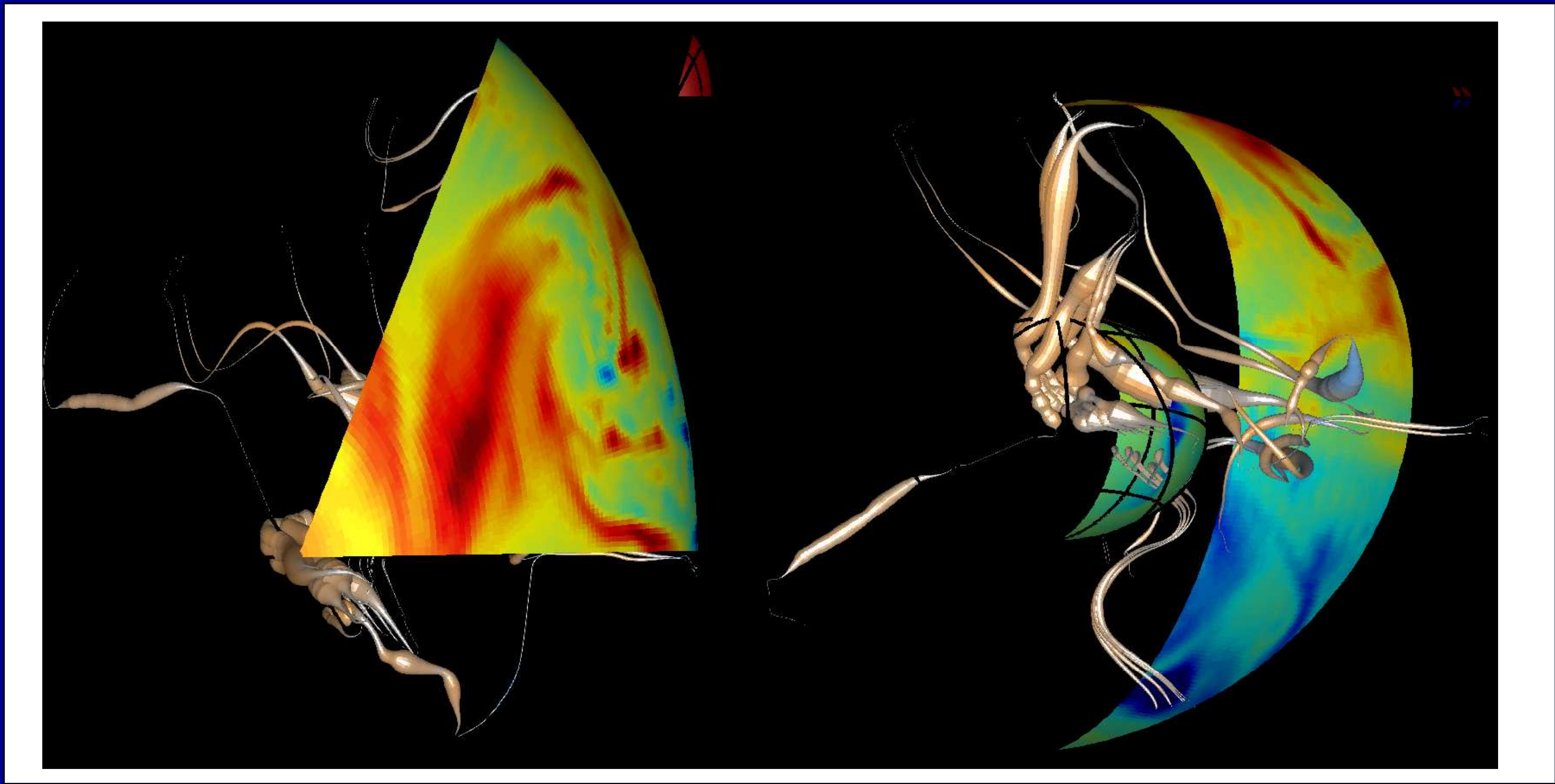

MAGNETIC ENERGY FOR E = 10^-5
1e+07
8e+06
6e+06
4e+06
2e+06
0
5000
10000
15000
20000
25000
30000
E_m
R_a
P_m = 0.5
P_m = 0.25
P_m = 0.1

$\tau = 5$

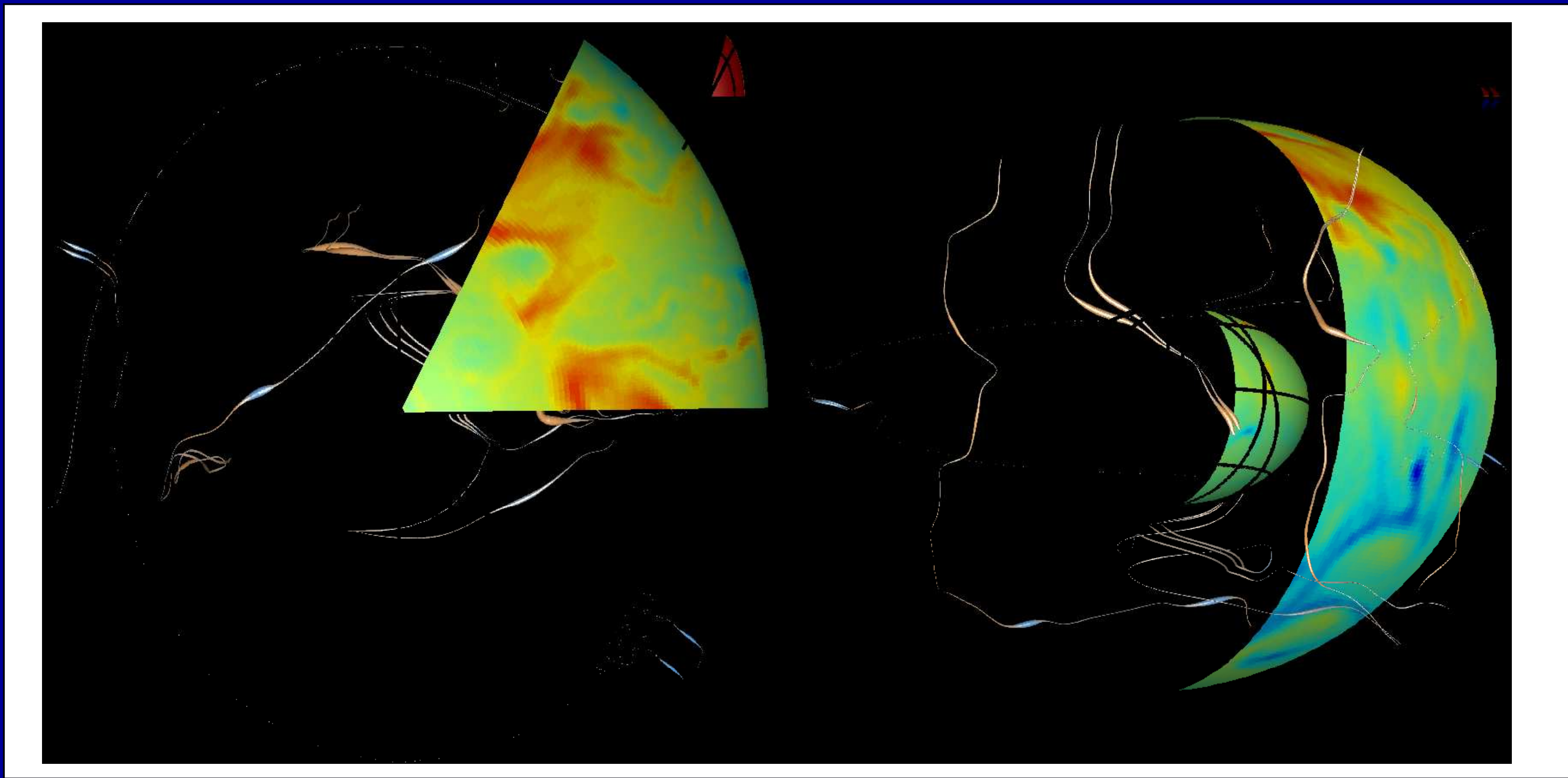
B_r **for** $R_a = 8000$ **and** $P_m = 0.5$



B_r **for** $R_a = 8000$ **and** $P_m = 0.25$



B_r **for** $R_a = 8000$ **and** $P_m = 0.1$



Rossby number

$E = 10^{-4}$	$R_a = 1550$
$R_o(P_m = 1)/R_o(P_m = 0.75)$	**0.98**
$R_o(P_m = 1)/R_o(P_m = 0.5)$	**0.94**
$R_o(P_m = 0.75)/R_o(P_m = 0.5)$	**0.97**

$E = 10^{-5}$	$R_a = 8000$
$R_o(P_m = 0.5)/R_o(P_m = 0.25)$	**0.87**
$R_o(P_m = 0.5)/R_o(P_m = 0.1)$	**0.71**
$R_o(P_m = 0.25)/R_o(P_m = 0.1)$	**0.81**

Conclusions - P_r and P_m

- for $P_r \geq 1$ and $P_m \geq 1$ the inertia is weak, dynamos are mostly dipolar and large-scale flows are columnar (e.g., *Christensen and Aubert (2006)*)

- for $P_r < 1$ the inertia becomes important but it depends on P_m. For $P_r < 1$ and $P_m \geq 1$ (rather $P_m \gg P_{m_{min}}$) is similar to $P_r \geq 1$ and $P_m \geq 1$, i.e. the inertia is again weak and dynamos are mostly dipolar and large-scale flows are columnar, while for $P_r < 1$ and $P_m < 1$ (rather $P_m \leq P_{m_{min}}$) the inertia becomes important and it is possible to observe the breakdown of the columnar structure of the convection in consequence of the dipolar structure breaks generally down. Fluid motion becomes strong in the polar regions and the magnetic field is convected out of polar regions (*Sreenivasan and Jones (2006)*)

- *Busse and Simitev (2005)* observed at low values of P_m a transition to hemispherical dynamos and at even lower values of P_m a further transition to quadrupolar dynamos (stress-free boundary conditions). These transitions are observed neither in *Sreenivasan and Jones (2006)* (no-slip boundary conditions, $P_m = P_r = 1, 0.5, 0.2$) nor in our study (no-slip boundary conditions)

- dipolar dynamo breaks down in *Sreenivasan and Jones (2006)* for $P_m = P_r = 0.2$ ($P_m < P_{m_{min}}$), while our magnetic field is dipolar and does not weaken because our P_m is greater or close to the value of $P_{m_{min}}$. The inertia becomes important but our dynamos remain dipolar

- it is possible to conclude that P_m governs a measure of inertia for low P_r, i.e. a measure of inertia depends on how far we are from $P_{m_{min}}$

- the real Earth's core - $E = 10^{-15}$, $P_m = 10^{-7}$, $P_r = 0.2$, $P_{m_{min}} \simeq 2.5 \times 10^{-9}$, i.e. the geodynamo works in the mode $P_m \gg P_{m_{min}}$

Šimkanin, J. and Hejda P., 2011. Hydromagnetic dynamos in rotating spherical fluid shells in dependence on the Prandtl number and stratification. *Geophys. J. Int.*, **185**, 637–646

Šimkanin, J. and Hejda P., 2011. Magnetic fields generated by hydromagnetic dynamos at the low Prandtl number in dependence on the Ekman and magnetic Prandtl numbers. *Earth Planet. Sci. Lett.*, submitted

THANK YOU


"Do, or do not. There is no 'try'."
- Yoda ('The Empire Strikes Back')
Bavaria
Bavaria...
It's Relentless