# Research of Earth rotation - past and present 

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Abstract: The rotation of the Earth is described by several motions:

- Motion of the Earth's spin axis in terrestrial frame (polar motion) $x, y$;
- Motion of the Earth's spin axis in space (precession/nutation) $\Delta \varepsilon, \Delta \psi$, or their difference from an adopted model, celestial pole offsets;
- Proper rotation around the spin axis, defined by Universal time UT1, or its difference from the International Atomic time TAI.
These five Earth Orientation Parameters (EOP) are necessary to calculate the transformation between celestial and terrestrial reference frames - see Fig. 1.


Fig. 1. Earth Orientation Parameters

The basic mathematical expressions that solve the orientation of the non-rigid Earth are Liouville equations, expressed in the rotating terrestrial frame:

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} t}(\boldsymbol{C} \boldsymbol{\omega}+\boldsymbol{h})+\omega \times(\boldsymbol{C} \boldsymbol{\omega}+\boldsymbol{h})=\boldsymbol{L} \tag{1}
\end{equation*}
$$

where $\boldsymbol{C}$ is the tensor of inertia (changing due to both external torques and transfer of mass inside the Earth), $\omega$ vector of rotation, $\boldsymbol{h}$ relative angular momentum (due to the motion of mass) and $\boldsymbol{L}$ external torque (exerted by the Moon, Sun and planets). The solution of Liouville equations yields the vector $\omega$ which, in turn, defines polar motion $\left(x=\omega_{1} /|\boldsymbol{\omega}|, y=-\omega_{2} /|\boldsymbol{\omega}|\right)$ and

[^0]relative change of the speed of rotation $\left(\omega_{3} /|\omega|\right)$. The motion of spin axis in space (angles $\theta, \psi$ ) and the angle of proper rotation $\varphi$ around the axis in space are then given as the solution of Euler kinematical equations
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$$
\begin{gather*}
\dot{\psi} \sin \theta=-\omega_{1} \sin \varphi-\omega_{2} \cos \varphi \\
\dot{\theta}=-\omega_{1} \cos \varphi+\omega_{2} \sin \varphi  \tag{2}\\
\dot{\varphi}=\omega_{3}-\dot{\psi} \cos \theta
\end{gather*}
$$
\]

The detailed inspection of Eqs. (1), (2) reveals that the motion of mass inside the Earth and near its surface (they are long-periodic in terrestrial frame) dominantly influence polar motion and speed of rotation, while the external torques (long-periodic in celestial frame) have dominant effect, both in precession-nutation and speed of rotation.
In the past, Earth orientation was monitored by the methods of optical astrometry; when modern observation techniques of space geodesy (SLR, VLBI, GPS) appeared, they replaced all existing services in the newly established International Earth Rotation and Reference Systems Service (IERS, founded in 1987).
The observations gathered by different techniques during more than a century serve as a basis for research, in which all five EOP are studied. The observed values are confronted with the values predicted by theory, and explanations of the differences are looked for.
Here follows a short summary of the results obtained so far:

1. Polar motion. There are two main periodic components - free (Chandler) wobble, with period of about 435 days and annual wobble with period of 365 days, both with amplitude of about several meters at the Earth's surface. The first one is almost circular prograde motion whose amplitude and phase are not stable. Theoretically, for the Earth model with visco-elastic mantle, liquid core and oceans, the phase should be constant and amplitude exponentially decreasing in time. Thus there must have been an excitation responsible for these phenomena. In modern times, there are excitation functions available for the atmosphere and oceans. For the past data we can only speculate about the character of the excitation that could cause the observed changes. The prograde annual wobble is slightly elliptic and much more stable; it is ascribed to atmospheric and oceanic forcing. A small part is probably caused by hydrologic
excitations, which are however not well known from independent sources. There also exists a recent secular motion towards Greenland (of about ten meters per century), most probably a part of a very long-periodic motion, linked with recent deglaciation accompanied by a slow change of tensor of inertia. The effect of strong earthquakes is theoretically predictable as a sudden step in the path of the pole, but it is so small (some centimeters) that it is hardly detectable by modern techniques.
2. Proper rotation. There is a wide spectrum of frequencies with which the speed of rotation changes, with dominant annual and semi-annual terms. These variations are mostly ascribed to tidal changes of the principal moment of inertia and atmospheric/oceanic excitations. There exist also decadal variations (probably caused by the changes on the core-mantle boundary) and gradual deceleration (due to tidal friction and secular change of the principal moment of inertia). Again, stepwise changes of speed of rotation caused by large earthquakes can be calculated, but very small to be detected by observation.
3. Precession-nutation. These motions are dominantly affected by the external torques exerted by the Moon, Sun, and also planets. Since the motion of the bodies of the solar system is predictable with a high accuracy, these effects can be modeled, taking into account also the reaction of the non-rigid Earth to external forces. Small deviations from the accepted model, celestial pole offsets, are derived from the observations. There is a free motion with period of about 430 days in celestial frame and amplitude of about $100 \mu$ arcseconds due to the liquid outer core of the Earth (Free Core Nutation, FCN), and evident annual and semi-annual changes with similar amplitudes, caused by atmospheric/oceanic excitations. These excitations are very small and near-diurnal in terrestrial frame, but their effect is multiplied by the resonance near FCN.

Key words: Earth rotation, celestial mechanics, geophysical excitations


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